2nd Taiwan-Japan Joint Workshop on Nonlinear Analysis and Optimization

Niigata University and Nagaoka University

Auguet 3-5, 2026

Organized by

Jein-Shan Chen, National Taiwan Normal University, Taiwan

Wei-Shih Du, National Kaohsiung Normal University, Taiwan

Tone-Yau Huang, Feng Chia University, Taiwan

Yutaka Saito, Niigata University, Japan

Kazutaka Sakai, Nagaoka University, Japan

Tamaki Tanaka, Niigata University, Japan

Syuuji Yamada, Niigata University, Japan

第２回非線形解析と最適化に関する日台共同研究集会
The 2nd Taiwan-Japan Joint Workshop on Nonlinear Analysis and Optimization (TJWNAO 2026)

The 2nd Taiwan-Japan Joint Workshop on Nonlinear Analysis and Optimization (TJWNAO2026) is held at both Niigata University and Nagaoka University, Japan, from August 3rd to 5th, 2026. The purpose of this international conference is to continue and deepen the friendships between Taiwanese and Japanese researchers and to exchange mathematical ideas and recent topics on nonlinear analysis and optimization. **The conference is supported by National Science and Technology Council (Taiwan), Niigata Visitors & Convention Bureau, Nagaoka and Niigata Universities.**

Organizers　Jein-Shan Chen (National Taiwan Normal University, Taiwan), Wei-Shih Du (National Kaohsiung Normal University, Taiwan), Tone-Yau Huang (Feng Chia University, Taiwan), Yutaka Saito (Niigata University, Japan), Kazutaka Sakai (Nagaoka University, Japan), Tamaki Tanaka (Niigata University, Japan), Syuuji Yamada (Niigata University, Japan)

記

日　時：　2026 年8 月 3 日（月）　August 3 (Mon)～
　　　　　8 月 5 日（水）　August 5 (Wed)

場　所：　長岡大学 Nagaoka University (Aug.3)，新潟大学 Niigata University (Aug.4–5)
　　Nagaoka Univ.　80-8 Oyama-machi, Nagaoka city 940–0828, Japan;
　　Niigata Univ.　8050 Ikarashi 2-no-cho, Nishi-ku, Niigata city 950–2181, Japan

プログラム
— PROGRAM —

August 3 (Mon)

Place: Nagaoka University

Opening Address, 13:30– (Prof. Tamaki Tanaka on behlf of the Organizers)

Invited Presentations M1 (Chairperson: Tamaki Tanaka)

13:45–14:15　**Generalize Dancš-Hegedüs-Medvegyev's Principle with Applications**
Lai-Jiu Lin (National Changhua University of Education, Taiwan)
林來居（台湾・国立彰化師範大学）

14:15–14:45　**A Representation of Approximation Functions as Convolutional Neural Networks and Probing the Corresponding Extension**
Chien-Chang Yen (Fu Jen Catholic University, Taiwan)
嚴健彰（台湾・輔仁大学）

(Tea break)

Invited Presentations M2 (Chairperson: Kazutaka Sakai)

15:00–15:30　**Semicontinuity of Composite Function via the Inheritance Mechanism on the Semicontinuity of Set-Valued Maps**
Tamaki Tanaka (Niigata University, Japan)
田中　環（日本・新潟大学）

15:30–16:00　**Optimal Policy Problem of Bipedal Walker Environment in OpenAI Gymnasium using Deep Reinforcement Learning**
Tone-Yau Huang (Feng Chia University, Taiwan)
黃同瑤（台湾・逢甲大学）

(Nagaoka Fireworks Festival 19:20– for interested persons)

August 4 (Tue)

Place: the Library Hall at Niigata University

Opening Address, 10:20– Speeches: Dean Prof.Eiichi Takazawa, Dean Prof.Jein-Shan Chen,
Niigata Tourism Goodwill Ambassador (にいがた観光親善大使)

Invited Presentations T1 (Chairperson: Tamaki Tanaka)

10:50–11:20 **Smoothing Functions for Sparse Optimization: A Unified Framework**
Jein-Shan Chen (National Taiwan Normal University, Taiwan)
陳界山（台湾・国立台湾師範大学）

11:20–11:50 **A Global Optimization Algorithm for Reverse Convex Quadratic Programming Problems and Its Applications**
Syuuji Yamada (Niigata University, Japan)
山田修司（日本・新潟大学）

Photo Session (all participants together on the stage after the morning session T1)

(Lunch) Invited speakers and organizers can take a lunch box in the meeting room of the library.

Invited Presentations T2 (Chairperson: Jein-Shan Chen)

13:30–14:00 **Variational Framework for Distortion Control in Surface Parameterizations**
Mei-Heng Yueh (National Taiwan Normal University, Taiwan)
樂美亨（台湾・国立台湾師範大学）

14:00–14:30 **The Role of Angiogenesis in Mathematical Modeling of Triple-Negative Breast Cancer**
Hsiu-Chuan Wei (Feng Chia University, Taiwan)
魏秀娟（台湾・逢甲大学）

14:30–15:00 **A Method for Establishing an Optimal Workforce Dispatching Model Balancing Both Psychological and Physiological Conditions**
Chung-Chien Hong (National Pingtung University of Science and Technology, Taiwan)
洪宗乾（台湾・国立屏東科技大学）

(Tea break)

Invited Presentations T3 (Chairperson: Tone-Yau Huang)

15:15–15:45 **Shrinking Projection Methods with Extended Allowable Ranges for a Family of Quasi-Nonexpansive Type Mappings**
Takanori Ibaraki (Yokohama National University, Japan)
茨木貴徳（日本・横浜国立大学）

15:45–16:15 **Solving Quadratic Unconstrained Binary Optimization by Digital and GPU Annealers with AI**
Yu-Chen Shu (National Cheng Kung University, Taiwan)
舒宇宸（台湾・国立成功大学）

16:15–16:45 **New Set-Valued Ekeland's Variational Principle for Essential Distances with Applications**
Wei-Shih Du (National Kaohsiung Normal University, Taiwan)
杜威仕（台湾・国立高雄師範大学）

Leaving at 17:00, we will go to Hot spring "Jonnobi" and take dinner and Spa for 17:30–19:45. A hotel shuttle bus will depart to the hotel from "Jonnobi" at 20:00.

August 5 (Wed)

Place: Large Seminar Room (5th floor) in the Science Building at Niigata University

Invited Presentations W1 (Chairpersons: Jen-Yen Lin and Der-Chiang Li)

9:30–10:00 **Some Algorithms for Generalized Inverse Mixed Variational Inequalities**
Chih-Sheng Chuang (National Chiayi University, Taiwan)
莊智升（台湾・国立嘉義大学）

10:00–10:30 **An Entropy-Weighted Local Intensity Clustering Model for Image Segmentation under Intensity Inhomogeneity**
Cheng-Shu You (Feng Chia University, Taiwan)
游承書（台湾・逢甲大学）

10:30–11:00 **Strong Convergence Theorems for Zero Points of Maximal Monotone Operators and its Applications**
Shunsuke Kajiba (Teikyo University, Japan)
梶葉駿介（日本・帝京大学）

(Tea break)

Invited Presentations W2 (Chairperson: Takanori Ibaraki)

11:15–11:45 **Exploring the Impact of Structural Dysconnectivity on Cognitive Computation: A Connectome-informed Reservoir Computing Study of Schizophrenia**
Ti-Wen Lu (National Cheng Kung University, Taiwan)
盧提文（台湾・国立成功大学）

11:45–12:15 **Matrix Optimization Problems with Maximum-Norm Constraints and Their Application to Finite Element Interpolation Error Estimates**
Xuefeng Liu (Tokyo Woman's Christian University, Japan)
劉雪峰（日本・東京女子大学）

(Lunch) The 1st student restaurant (第一食堂) or LAWSON near the library are available. See the map.

Invited Presentations W3 (Chairperson: Xuefeng Liu)

13:30–14:00 **A Closed-Form Distributionally Robust Decision Rule under Conformally Calibrated Wasserstein Ambiguity Sets**
Bin-Wei Hsu (National Pingtung University of Science and Technology, Taiwan)
徐彬偉（台湾・国立屏東科技大学）

14:00–14:30 **Cone-Order Regression from Preference Data and Existence of its Solutions**
Yutaka Saito (Niigata University, Japan)
齋藤　裕（日本・新潟大学）

14:30–15:00 **Numerical Experiments for Estimating Set Inequalities via Sublinear Set Functionals with SetOptTools**
Yuto Ogata (Niigata University, Japan)
小形優人（日本・新潟大学）

(Tea break)

Invited Presentations W4 (Chairperson: Yuto Ogata)

15:10–15:40 **A Minimax Theorem for Set-Valued Maps and its Application to Multi-Objective Uncertainty**
Yousuke Araya (Akita Prefectural University, Japan)
荒谷洋輔（日本・秋田県立大学）

15:40–16:05 **Proximal point algorithm on geodesic spaces with a new resolvent operator**
Takuto Kajimura (Toho University, Japan)
梶村拓豊（日本・東邦大学）

16:05–16:30 **A Framework of Smoothing Penalty Methods for Convex Second-Order Cone Programming: Fundamentals and Application**
Ryota Iwamoto (National Taiwan Normal University, Taiwan; Niigata University, Japan)
岩本崚汰（台湾・国立台湾師範大学，日本・新潟大学）

Closing Address, 16:30–16:35

[Place & Transportation on Aug.3 (Mon)]

The conference site is Nagaoka University (長岡大学), and its campus access information:

`https://www.nagaokauniv.ac.jp/access/`

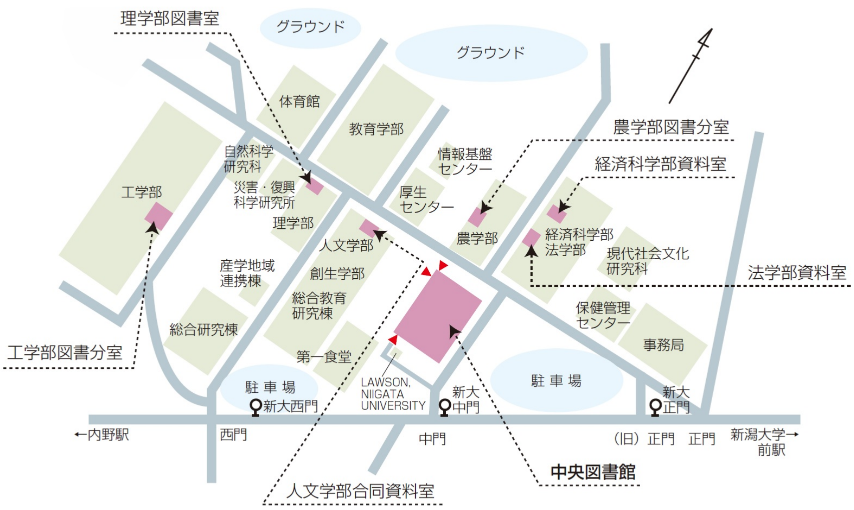
Between the East Exit at Nagaoka JR station and Nagaoka University, there is a bus, which time table is referred at the following webpage (in Japanese)

`https://www.echigo-kotsu.co.jp/contents/diagram/route/nagaoka/east.html#bus-stop-map`

Please go to No.2 bus stop at the East Exit of Nagaoka JR station, take a bus, and then the bus will arrive at its destination in 15 minutes. The bus runs every 20 minutes.

[Place & Transportation on Aug.4 (Tue)]

The conference site is Library Hall in the central Library (中央図書館) at Niigata University (新潟大学), and its campus access information: `https://www.lib.niigata-u.ac.jp/about/access/`



All participants can take the hotel bus (Terminal ArtInn) to come to the campus if they can reach the hotel by **9:10 on Aug.4 (Tue)**; **this shuttle bus departs at 9:15**. Otherwise please take a train (JR Echigo-line, 越後線) or a city bus (Niigata-kotsu) to reach the Ikarashi-Campus. In case of train, please get off at "Niigata-Daigakumae station (新潟大学前駅)" and take a walk in 15 minutes. In case of bus, please go to No.7 bus stop at Niigata JR station, take a bus (W2: bound for Niigata University), and then please get off at "Shinda-Nakamon (新大中門)" in 43 minutes. The bus runs every 15–20 minutes.

[Dinner & Spa in the evening of Aug.4 (Tue)]

By Niigata University bus at 17:00, we will go to Hot spring "Jonnobi" and take dinner and Spa for **17:30–19:45**. The hotel bus departs to the hotel from "Jonnobi" at 20:00. Don't miss it!

[Place & Transportation on Aug.5 (Wed)]

The conference site is the **Lagest seminar room of Dept.Math. (5th floor)** in Science Building (理学部) at Niigata University (新潟大学), and its campus access information in the map above. All participants can take the hotel bus (Terminal ArtInn) to come to the campus if they can reach the hotel by **8:10 on Aug.5 (Wed)**; **this shuttle bus departs at 8:15**. Otherwise please take a train (JR Echigo-line, 越後線) or a city bus (Niigata-kotsu) to reach the Ikarashi-Campus. In case of train, please get off at "Niigata-Daigakumae station (新潟大学前駅)" and take a walk in 20 minutes. In case of bus, please go to No.7 bus stop at Niigata JR station, take a bus (W2: bound for Niigata University), and then please get off at "Shinda-Nishimon (新大西門)" in 45 minutes. The bus runs every 15–20 minutes.

[Banquet in the evening of Aug.5 (Wed)]

The hotel shuttle bus to the hotel departs at 16:50. All participants are welcome to the **Banquet at 海鮮家 "葱ぼうず (Negi-bouzu)" near Hotel Terminal ArtInn** which starts **18:30–** :
`https://www.yonekura-group.jp/shop/negibouzu/plan.html` with Banquet fee JPY6,000.

GENERALIZE DANCŠ-HEGEDÜS-MEDVEGYEV'S PRINCIPLE WITH APPLICATIONS

LAI-JIU LIN*, SUNG-YU WANG
NATIONAL CHANGHUA UNIVERSITY OF EDUCATION IN TAIWAN

We establish an existence theorem for stationary points with a common fixed point for set-valued mappings by using sizing-up functions, under assumptions weaker than reflexivity and transitivity. Our result extends the Dancš–Hegedüs–Medvegyev principle [4] and related results of Lin–Du[12] and [16]. As applications, we obtain fixed point theorems of Subrahmanyam type [21], solvability results for variational relation and variational inclusion problems [2, 10, 14, 15, 17], a simultaneous quasi-Ekeland type variational principle [11], and existence results for Pareto-type optimization problems.

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A REPRESENTATION OF APPROXIMATION FUNCTIONS AS CONVOLUTIONAL NEURAL NETWORKS AND PROBING THE CORRESPONDING EXTENSION

CHIEN-CHANG YEN
MATHEMATICS, FU-JEN CATHOLIC UNIVERSITY, TAIWAN

A function can be representation as the linear combination or a non-linear composition by a set of functions, for instance Taylor expansion, Fourier series, and Kolmogrov-Arnold representation. In this decade, the artificial neural networks using activation functions and bias are popular on large language models (LLM), small language models (SLM), and physics informed neural networks. The universe approximation theory has been used on showing the networks which approximate the continuous functions. However, there still a lot of mystery on the effects of a neural networks. The interpretation of a neural network compared with the traditional function approximation is interested in this talk. The approach of approximation functions is regarded as convolutional neural networks (CNN). Meanwhile, the hidden layers represent to construct bases and the functional values appear in the last dense layer. Such CNN representation seems an extension of approximation by generalizing polynomials to be activation functions. Finally, we further probe the behavior of the effect of this extension using the Taylor expansion of functions.

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Taiwan-Japan Joint Workshop on Nonlinear Analysis and Optimization (TJWNAO2026)
Held in Nagaoka Univ. on Aug. 3 and Niigata Univ. on Aug. 4–5, 2026

SEMICONTINUITY OF COMPOSITE FUNCTION VIA THE INHERITANCE MECHANISM ON THE SEMICONTINUITY OF SET-VALUED MAPS

PREMYUDA DECHBOON, RYOTA IWAMOTO, TAMAKI TANAKA*, SYUUJI YAMADA
BURAPHA UNIVERSITY IN THAILAND, NIIGATA UNIVERSITY IN JAPAN

A composite function is a function which is the nesting of two or more functions to form a single new function. Such operation frequently preserves several mathematical properties of each nested function. For instance, a composition of continuous maps is continuous on topological spaces. From the view point of vector optimization and set optimization, this kind of inheritance by composite operations is important and useful to prove extended results and to get characterizations of optimal solutions through scalarization. This is a typical approach by which optimization problems with vector-valued or set-valued maps can be easily handled by converting vectors or sets into real numbers; see [3] and [4, 5].

Let X, Y, and C be a topological space, an ordered real topological vector space, and a convex ordering cone in Y, respectively. In 1996, Tanaka [12] proved the inheritation of the composition of a cone-continuous vector-valued function $f : X \to Y$ and any element of the dual cone $C^+_{X'}$, which consists of nonnegative continuous linear functions on a convex cone $C \subset X$. In 2006, Kimura and Tanaka [7] mentioned its generalized inheritation of the composition of a cone-continuous vector-valued function $f : X \to Y$ and the Gerstewitz's (Tammer's) nonlinear scalarizing functional:

$$h_C(v; d) := \inf \{t \in \mathbb{R} \mid v \in td - C\} \tag{1}$$

where C is a convex cone in a real vector space and $d \in C$. This scalarizing functional $h_C(\cdot; d)$ is sublinear (i.e., $h_C(v_1 + v_2; d) \leq h_C(v_1; d) + h_C(v_2; d)$ and $h_C(tv; d) = th_C(v; d)$ for $t > 0$) and hence this conversion is called "sublinear scalarization," found early in [2, 10]. If convex cone C is a half space, that is, $C = \{v \mid \langle w, v\rangle \geq 0\}$ with w satisfying $\langle w, w\rangle = 1$, then $h_C(v; w) = \langle w, v\rangle$. Therefore this special functional is a certain generalization of linear scalarization including the notions of weighted sum and inner product. Accordingly, this idea has inspired some researchers to develop particular scalarization methods for sets.

Recently, Ike, Liu, Ogata and Tanaka [6] show certain results on the inheritance property of some kinds of continuity of set-valued maps via scalarization functions for sets: if a set-valued map has a kind of continuity (lower continuity or upper continuity; see [4]) then the composition of its set-valued map and a certain scalarization function assures a similar semicontinuity to its scalarization function defined on the family of nonempty subsets of a real topological vector space. Their results are generalizations of results in earlier study by Kuwano, Tanaka and Yamada [9]. However, the statements of inheritance in [6] are confined to four types out of the six set-relations proposed by Kuroiwa, Tanaka and Ha [8]. On the other hand, Sonda, Kuwano, and Tanaka [11] introduce two kinds of continuity with respect to cone, called "cone continuity," for set-valued maps by analogy with semicontinuity for real-valued functions, and they investigate the inheritance properties on cone continuity of parent set-valued maps via scalarization. Therefore, it is interesting to investigate the inheritance of cone continuity for set-valued maps via general scalarization functions for sets in the same manner as [6].

The aim of this talk is to introduce the mechanism by which composite functions of a set-valued map and a scalarization function transmit semicontinuity of parent set-valued maps through several scalarization for sets; see [1].

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OPTIMAL POLICY PROBLEM OF BIPEDALWALKER ENVIRONMENT IN OPENAI GYMNASIUM USING DEEP REINFORCEMENT LEARNING

TONE-YAU HUANG
FENG-CHIA UNIVERSITY IN TAIWAN

In this talk, we will focus on deep reinforcement learning algorithms, introducing the Actor-Critic Method, Proximal Policy Optimization (PPO), and other related methods. Furthermore, we will investigate optimal policy generation using the BipedalWalker environment from OpenAI Gymnasium as our primary case study.

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DOI: 10.1109/IV48863.2021.9575954 (2021)

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SMOOTHING FUNCTIONS FOR SPARSE OPTIMIZATION: A UNIFIED FRAMEWORK

JEIN-SHAN CHEN
NATIONAL TAIWAN NORMAL UNIVERSITY IN TAIWAN

This talk presents a unified framework for constructing smoothing functions tailored to a broad class of widely used regularizers, including the plus function, the pinball function, the ℓ_0-norm, the ℓ_p-norm for $0 < p \leq 1$, the MCP, and the SCAD. By transforming nonsmooth regularizers into smooth approximations, the proposed framework facilitates the application of efficient optimization algorithms to sparse optimization problems. The framework is systematically derived from continuous approximations of the step function, offering a principled approach to generating smoothing functions across various regularizers. These approximations are, in turn, constructed using polynomial functions and the Dirac delta function.

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A GLOBAL OPTIMIZATION ALGORITHM FOR REVERSE CONVEX QUADRATIC PROGRAMMING PROBLEMS AND ITS APPLICATIONS

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VARIATIONAL FRAMEWORK FOR DISTORTION CONTROL IN SURFACE PARAMETERIZATIONS

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Surface parameterization establishes a coordinate correspondence between a surface and a pla-nar or canonical domain. This provides coordinates in which surface-processing tasks can be carried out in a simpler computational setting. The usefulness of such parameterizations, how-ever, depends on controlling induced angle, area, and metric distortions. In this talk, I will present a variational framework for distortion control in surface parameterizations. The discus-sion will focus on energy formulations underlying conformal [1], authalic (area-preserving) [2], and distortion-balancing parameterizations [3]. I will also discuss how these formulations lead to practical numerical optimization algorithms. Applications to surface registration and effective surface representations will be demonstrated to illustrate the practical utility of these methods.

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THE ROLE OF ANGIOGENESIS IN MATHEMATICAL MODELING OF TRIPLE-NEGATIVE BREAST CANCER

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In this study, a mathematical model of triple-negative breast cancer (TNBC) is proposed. The mathematical model includes interactions between tumor cells, innate and adaptive immune cells, and the angiogenic factor. Without the angiogenic factor, the mathematical model exhibits three coexisting stable equilibria, representing the tumor-free equilibrium (immune surveillance), the microscopic tumor equilibrium (equilibrium phase), and the large-tumor equilibrium (immune escape). However, the microscopic tumor equilibrium can break down when the angiogenic factor is included. The tumor may start to grow larger depending on how rapidly the angiogenic factor increases, which indicates how quickly new blood vessels are recruited to nourish the tumor. The microscopic tumor equilibrium can last for months, years, or even a lifetime. Experimental data in three independent studies are used to estimate the parameter values. The parameter values are consistent across all three studies. Further estimation of parameter distributions using the MCMC method is performed to study individual variation. Mathematical analysis, such as the nonnegativity and boundedness of solutions as well as stability analysis of the equilibria, is conducted. In addition, global sensitivity analysis and bifurcation analysis are investigated. The results of the numerical simulations are compared with the observed medical phenomena. The model can serve as a foundation for further investigation of angiogenesis-related treatments and other therapeutic approaches, including immunotherapy and chemotherapy.

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Key words and phrases. Mathematical modeling, Triple-negative breast cancer, TNBC, Angiogenesis, Immunoediting, Bifurcation analysis.

A METHOD FOR ESTABLISHING AN OPTIMAL WORKFORCE DISPATCHING MODEL BALANCING BOTH PSYCHOLOGICAL AND PHYSIOLOGICAL CONDITIONS

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Labor shortages have become increasingly severe in Taiwan, particularly within traditional industries. In shift-based operational environments, the design of workforce dispatching and job rotation systems plays a critical role in balancing employees' psychological and physical workloads. Achieving such balance is essential for reducing turnover, improving operational stability, and enhancing organizational competitiveness.

This study investigates a chemical manufacturing plant in southern Taiwan operating under a three-shift system, where each shift involves multiple tasks with varying mental and physical demands. Imbalanced task assignments may lead to worker fatigue and inefficiencies, which contradict the principles of lean management. To address this issue, this research integrates multiple decision-making and optimization methods. First, the Analytic Hierarchy Process (AHP) is employed to capture shift workers' perceptions of workload across different shifts and task types. The aggregated results are then used, in conjunction with managerial input, to construct alternative dispatching models. These models are treated as controllable factors and evaluated using the Taguchi method. Subsequently, the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) is applied to identify the optimal dispatching model, with psychological and physical workload balance defined as dual optimization objectives.

The results indicate that the proposed model effectively improves the balance of psychological and physical workloads among shift workers. Sensitivity analysis further confirms the robustness and reliability of the model. The findings provide practical guidance for workforce scheduling in shift-based industrial settings.

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SHRINKING PROJECTION METHODS WITH EXTENDED ALLOWABLE RANGES FOR A FAMILY OF QUASI-NONEXPANSIVE TYPE MAPPINGS

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Fixed point approximation methods for nonexpansive mappings play an important role in nonlinear analysis and its applications. One useful method is the shrinking projection method introduced by Takahashi, Takeuchi, and Kubota [9]. Since then, many researchers have studied and generalized this method. In particular, Kimura and Takahashi [6] improved the method by using the concept of Mosco convergence and established strong convergence theorems for relatively nonexpansive mappings in Banach spaces.

However, the shrinking projection method requires the exact computation of metric projections at each iteration step, and this computation becomes increasingly difficult as the iteration proceeds. To solve this problem, Kimura [3] proposed the shrinking projection method with nonsummable errors, which allows errors inside the target sets; see also [4,5]. Inspired by Kimura [3], Takeuchi [10] introduced the shrinking projection method with allowable ranges, which also allows errors outside the target sets.

On the other hand, in the study of common fixed point approximation methods, Ibaraki and Takeuchi [2] extended the concept of quasi-nonexpansive mappings, which generalize nonexpansive mappings, to families of nonlinear mappings by using the notion of attractive points. Furthermore, Lin and Takahashi [7] extended the concept of attractive points from Hilbert spaces to Banach spaces.

Motivated by these works, we propose a new shrinking projection method that allows errors both inside and outside the target sets. Using this method, we establish strong convergence theorems for finding a common fixed point of a family of nonlinear mappings of quasi-nonexpansive type in Banach spaces [1].

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SOLVING QUADRATIC UNCONSTRAINED BINARY OPTIMIZATION BY DIGITAL AND GPU ANNEALERS WITH AI

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In this talk, I will delve into the combinatorial optimization problems formulated in the quadratic unconstrained binary optimization (QUBO) framework and solve them by the digital and GPU annealers. I will present a comparative analysis of digital annealers versus traditional annealers, focusing on their performance across a range of classical NP-hard problems. Additionally, I will show some recent results about the AI predictions for QUBO initialization.

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NEW SET-VALUED EKELAND'S VARIATIONAL PRINCIPLE FOR ESSENTIAL DISTANCES WITH APPLICATIONS

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The celebrated Ekeland's variational principle (see [5, 6]), established in 1972 for characterizing approximate solutions to nonconvex minimization problems in metric spaces, has emerged as a cornerstone of nonlinear functional analysis and applied mathematics. Benefiting from its profound theoretical significance and versatility, Ekeland's variational principle has spurred extensive research into its generalizations in diverse directions; see, for instance, [1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] and the references therein. It is well known that the primitive Ekeland's variational principle is equivalent to the Caristi's fixed point theorem and to the Takahashi's nonconvex minimization theorem (for more details, see, e.g., [9, 11, 12, 14, 16]).

In this talk, we present new versions of the generalized Ekeland's variational principle for set-valued mappings and essential distances (see, e.g., [3, 4]) in the setting of (L, K)-lower complete metric spaces (see, e.g., [14]). These results are then applied to establish some new Caristi type fixed point theorems, Takahashi type nonconvex minimization theorems and others.

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SOME ALGORITHMS FOR GENERALIZED INVERSE MIXED VARIATIONAL INEQUALITIES

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We study the generalized inverse mixed variational inequality (GIMVI) problem in real Hilbert spaces. Unlike many existing works, we do not assume that (F, G) is a strongly monotone couple. Instead, we assume that each operator is strongly monotone and Lipschitz continuous. We propose one proximal gradient method and two proximal extragradient-type methods, and show strong convergence under suitable conditions.

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AN ENTROPY-WEIGHTED LOCAL INTENSITY CLUSTERING MODEL FOR IMAGE SEGMENTATION UNDER INTENSITY INHOMOGENEITY

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In this talk, we present an entropy-weighted local intensity clustering model for image segmentation under intensity inhomogeneity induced by bias fields. The model is formulated in a variational framework by minimizing an energy functional that combines boundary regularization with a data-fitting term for image partitioning. The boundary length is approximated using heat kernel convolution on the characteristic functions of the segmented regions. The data-fitting term is derived from the multiplicative bias field resulting in a local intensity clustering property and further weighted by the local entropy. Furthermore, the model is implemented through an iterative convolution-thresholding scheme with provably guaranteed energy decay. Finally, We present numerical simulations to demonstrate the effectiveness and superior performance of the proposed approach.

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STRONG CONVERGENCE THEOREMS FOR ZERO POINTS OF MAXIMAL MONOTONE OPERATORS AND ITS APPLICATIONS

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In 2008, Takahashi, Takeuchi and Kubota [7] introduced the shrinking projection method which is an iterative method for finding a common fixed point of some families of nonlinear mappings in a Hilbert space. In 2009, Kimura and Takahashi [5] improved this method in a Banach space. In the shrinking projection method, we need to obtain the exact value of the metric projection to generate a sequence in every step, which is a difficult task. To solve this problem, in 2012, Kimura [4] proposed the shrinking projection method with nonsummable errors in a geodesic space. Motivated by Kimura [4], in 2019, Takeuchi [6] presented another method which is called the shrinking projection method with allowable ranges. They obtained a strong convergence theorem for finding a fixed point of a nonlinear mapping in a Banach space.

On the other hand, the shrinking projection method is also used to establish convergence theorems for finding a zero point. In 2009, by using the original shrinking projection method, Inoue, Takahashi and Zembayashi [3] obtained a strong convergence theorem for finding a zero point of a maximal monotone operator in a Banach space. In 2016, Ibaraki [1] studied the shrinking projection method with nonsummable errors introduced by Kimura [4] for the zero point problem in a Banach space. In 2025, Ibaraki and Kajiba [2] studied the shrinking projection method with allowable ranges introduced by Takeuchi [6] for the zero point problem in a Banach space.

In this talk, we study the shrinking projection method with allowable ranges by using the generalized projection for the zero point problem. The generalized projection is a different type of projection from that used by Ibaraki and Kajiba [2]. Using this method, we obtain strong convergence theorems for finding a zero point of a maximal monotone operator in a Banach space. Moreover, using our results, we consider several applications such as the convex minimization problem and the variational inequality problem.

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EXPLORING THE IMPACT OF STRUCTURAL DYSCONNECTIVITY ON COGNITIVE COMPUTATION: A CONNECTOME-INFORMED RESERVOIR COMPUTING STUDY OF SCHIZOPHRENIA

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The mapping between brain structure and functional dynamics remains a fundamental challenge in computational neuroscience. Schizophrenia is increasingly conceptualized as a "dysconnectivity syndrome," characterized by distributed alterations in neural wiring and a decreased capacity for functional integration. Recent neuroimaging studies using graph theory have identified a specific "affected core" within the schizophrenic connectome—a set of regions, including prefrontal and subcortical areas, that drive the loss of global network efficiency. This study utilizes Connectome-informed Reservoir Computing (RC) to explore how these specific topological impairments influence cognitive task performance. By integrating human connectome data into the RC framework, our aim is to simulate how the structural degradation of the "affected core" shifts the system's dynamical regimes and impacts its ability to process memory-related tasks. Preliminary results reveal no significant difference in memory capacity when evaluated at the whole-brain scale. This finding strongly suggests that global network dynamics may dilute localized deficits, highlighting the necessity for advanced matrix preprocessing and more targeted topological investigations.

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MATRIX OPTIMIZATION PROBLEMS WITH MAXIMUM-NORM CONSTRAINTS AND THEIR APPLICATION TO FINITE ELEMENT INTERPOLATION ERROR ESTIMATES

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Matrix optimization problems with maximum-norm constraints appear naturally when one tries to obtain explicit and sharp constants in finite element error analysis. Based on the work of Galindo, Ike, and Liu [1], this talk discusses such a problem arising from the maximum-norm estimate for the linear Lagrange interpolation on a triangular element K. The target constant $C^L(K)$ is defined by

$$\left\|u - \Pi^L u\right\|_{\infty,K} \leq C^L(K)|u|_{2,K}, \qquad u \in H^2(K),$$

where $\Pi^L u$ denotes the linear Lagrange interpolation of u. Unlike the classical L^2- or H^1-error estimates, the use of the maximum norm in the left-hand side leads to a nonsmooth constraint that is difficult to treat directly.

Let

$$\lambda(K) := \inf_{u \in V^L(K)} \frac{|u|_{2,K}^2}{\|u\|_{\infty,K}^2}, \qquad V^L(K) := \{u \in H^2(K) \mid u(p_i) = 0,\ i = 1,2,3\}.$$

Then $C^L(K) = \lambda(K)^{-1/2}$. Using the Fujino–Morley finite element space $V_h^{\mathrm{FM}}(K)$, the corresponding finite dimensional problem becomes

$$\lambda_h = \min_{x \in \mathbb{R}^M} x^T A x, \qquad \text{subject to } \left\|\sum_{i=1}^{M} x_i \phi_i\right\|_{\infty,K} \geq 1,$$

where $A = (\langle \phi_i, \phi_j \rangle_h)$ is the stiffness matrix associated with the broken H^2 inner product.

To handle this constraint efficiently, we use the Bernstein representation of piecewise quadratic functions. The convex-hull property of Bernstein polynomials gives

$$\left\|\sum_{i=1}^{M} x_i \phi_i\right\|_{\infty,K} \leq \|Bx\|_\infty,$$

where B is the matrix mapping Fujino–Morley coefficients to Bernstein coefficients. This leads to the relaxed matrix optimization problem

$$\lambda_{h,B} = \min_{x \in \mathbb{R}^M} x^T A x, \qquad \text{subject to } \|Bx\|_\infty \geq 1.$$

Although this is still a nonconvex constrained problem, its optimal value can be obtained explicitly. Since A is positive definite, by taking $A = R^T R$ and putting $y = Rx$, the problem becomes

$$\lambda_{h,B} = \min_{y \in \mathbb{R}^M} y^T y, \qquad \text{subject to } \left\|BR^{-1}y\right\|_\infty \geq 1.$$

If b_i^T denotes the ith row of BR^{-1}, then

$$\lambda_{h,B} = \frac{1}{\max_i \|b_i\|_2^2}.$$

Equivalently, without explicitly computing the Cholesky factor, one has

$$\lambda_{h,B} = \frac{1}{\max(\mathrm{diag}(BA^{-1}B^T))}.$$

Thus the constrained optimization problem is converted into a matrix computation involving the diagonal entries of $BA^{-1}B^T$. Hence no nonlinear iterative optimization is required; the computation is reduced to sparse matrix assembly and linear solves. On the other hand, for

*Presenting author.

dense finite element meshes the dimensions of A and B become large, and the efficient evaluation of the required diagonal entries remains an important computational issue. As future work, we will improve the present method for large-scale matrix computations, especially for refined finite element meshes.

We also explain how this computable lower bound for λ_h gives a rigorous lower bound for $\lambda(K)$, and hence a rigorous upper bound for $C^L(K)$, by using the orthogonality of the Fujino–Morley projection. For uniform meshes, a sharper correction formula can be derived from scaling properties of similar triangles. Numerical examples show that the proposed approach gives sharp and efficient estimates. For instance, for the unit right isosceles triangle, the method yields

$$0.40432 \le C^L(K) \le 0.41596,$$

while a direct theoretical estimate gives only the much rougher upper bound 1.3712. Interval arithmetic can also be incorporated in the assembly and matrix evaluation steps; for mesh size 1/64, the validated upper bound is contained in

$$[\,0.4159516728,\ 0.4159516793\,].$$

These results indicate that maximum-norm constrained matrix optimization, combined with finite element structure and Bernstein polynomial techniques, provides an effective framework for rigorous interpolation error estimation.

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A CLOSED-FORM DISTRIBUTIONALLY ROBUST DECISION RULE UNDER CONFORMALLY CALIBRATED WASSERSTEIN AMBIGUITY SETS

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Distributionally robust decision-making requires uncertainty sets that are both statistically valid and operationally meaningful under demand uncertainty. We propose a feature-adaptive distributionally robust optimization framework in which the Wasserstein-2 ambiguity radius is calibrated using split conformal prediction [1, 2]. Specifically, the Wasserstein ambiguity radius is set equal to the split conformal quantile of calibration residuals, establishing a direct connection between finite-sample statistical validity and distributionally robust optimization [3].

Under a Gaussian reference distribution fitted via NGBoost [4] and an upper-tail shift approximation, the resulting formulation yields a tractable closed-form decision rule that decomposes into two structurally distinct components: a context-adaptive DRO adjustment and a conformal correction term. This decomposition reveals a separation of roles within the framework. Conformal calibration guarantees finite-sample coverage independently of distributional assumptions, while the DRO component provides a robustness-oriented interpretation under the Gaussian-reference approximation [5].

Several theoretical properties are established. We show that the finite-sample coverage guarantee is preserved under the proposed DRO construction, characterize how heavy-tailed demand attenuates sensitivity to the cost ratio, and derive an explicit measure quantifying the value of improved forecasting precision. The framework links statistical reliability, decision robustness, and predictive uncertainty within a unified optimization structure, following the prescriptive analytics paradigm of [6].

Empirical validation is conducted using 953,736 London Fire Brigade incident records. The proposed method adapts deployment decisions from 2.1 to 7.2 appliances depending on context, compared with a fixed deployment level of 4.1 under a conventional fixed-radius DRO approach [7], while reducing average deployment by approximately 20% at comparable coverage levels. Additional out-of-time validation on 304,858 incidents from 2015–2017 confirms that the coverage guarantee remains valid across a distinct historical period without parameter re-tuning.

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CONE-ORDER REGRESSION FROM PREFERENCE DATA AND EXISTENCE OF ITS SOLUTIONS

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In machine learning and optimization, the choice of a reward function or an evaluation function has a substantial influence on the obtained solutions and generated outputs. Recent methods such as reinforcement learning from human feedback (RLHF) and direct preference optimization (DPO) use human pairwise preference data to learn reward models, policies, or policy objectives. In many such approaches, preference relations are often represented through scalar rewards or probabilistic pairwise-comparison models [5]. On the other hand, in multiobjective and vector optimization, there are frameworks in which multidimensional objects are compared directly by means of cone orders or Pareto orders, without necessarily scalarizing them [6, 1].

The aim of this study is to estimate an order structure from human pairwise preference data in a regression-like manner. The distinctive point of our approach is the combination of a circular cone model, text-pair embeddings of the type used in DPO datasets, and regression-like estimation of the induced order structure. We consider a set of preference observations on a vector space,

$$\{(x, y) \in X \times X \mid x \succ y\},$$

and approximate the relation $\succ$ by a cone order $\leq_K$ induced by a circular cone K. In many order-theoretic formulations, preference data are interpreted as observations of an underlying consistent preference relation. The present study is motivated by situations in which such an interpretation is difficult, as in human pairwise comparison data, and in which the data may contain contradictions or the unconstrained cone model may admit only a degenerate solution, such as an excessively wide cone that reduces empirical loss but yields a less informative order structure. We therefore introduce penalty terms so that a compromise solution can be obtained even in such cases.

For a sample $x_i \succ y_i$ of pairwise comparison data, we define the difference vector

$$z_i = x_i - y_i \quad (z_i \neq 0).$$

We interpret the observation as being better explained when z_i belongs to an unknown cone K. A basic estimation problem is formulated as

$$\widehat{K} \in \operatorname*{arg\,min}_{K \in \mathcal{K}} \left[\sum_{i=1}^{N} d(z_i, K) + \lambda \Omega(K) \right],$$

where d is a loss function associated with the cone, Ω is a regularization term controlling the complexity or width of the cone, and $\lambda > 0$ is a regularization parameter.

As a basic model, this talk focuses on a circular cone determined by a central direction $c \in \mathbb{R}^n$ and an angular parameter $\alpha = \cos\theta$:

$$K(c, \alpha) = \{z \in \mathbb{R}^n \mid \langle z, c \rangle \leq \alpha \|z\|,\ \|c\| = 1,\ 0 < \alpha \leq 1\}.$$

In this model, the object to be estimated is a finite-dimensional parameter consisting of the central direction and the angle. Compared with directly estimating a general closed convex cone, this setting is more tractable for discussing existence, stability, and visualization. As background, this type of conic constraint is also related to standard frameworks of conic optimization [3]. For example, using the angular margin loss

$$\ell(z; c, \alpha) = \left[\max\left\{ 0, \alpha - \frac{\langle z, c \rangle}{\|z\|} \right\} \right]^2,$$

we can construct a cone-order regression problem in which the loss becomes smaller as the observed preference difference approaches the central region of the cone.

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We establish a basic existence result under compact parameter restrictions and continuous losses, and then discuss possible extensions toward stability, algorithmic convergence, and non-uniqueness. For instance, one may restrict the parameter space of (c, α) to

$$S^{n-1} \times [\varepsilon, 1] \quad (\varepsilon > 0)$$

and assume continuity of the loss function and the regularization term. Since uniqueness does not generally hold, we do not initially aim to prove global uniqueness. Instead, we discuss uniqueness from the viewpoints of isolated local solutions and uniqueness of the limit point for an initial region of an iterative algorithm.

In a validation phase, we plan to use DPO-style text-pair preference data as an example of pairwise preference observations, embed the texts into a vector space, estimate the circular cone parameters, and visualize the obtained central direction and angle. This visualization is expected to help explain latent evaluation axes in the dataset and possible biases of labelers. In this sense, the present study connects human preference learning represented by RLHF and DPO with multiple criteria decision-making research based on convex preference cones and cone contraction algorithms [2, 4].

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NUMERICAL EXPERIMENTS FOR ESTIMATING SET INEQUALITIES VIA SUBLINEAR SET FUNCTIONALS WITH SETOPTTOOLS

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In set optimization, various definitions and applications of set inequalities (e.g., [1, 2]) have been studied. However, there are still few challenging or compromise-oriented applications to practical problems. Although several researchers have actively investigated set-valued objects against such issues (e.g., [3, 4, 5, 6]), the vast majority of the literature focuses on theoretical problems and formal extentions.

Some of the reasons lie in the difficulty of handling infinite dimensional spaces and unbounded sets, which should be solved finitely and algorithmically. Another problem is the shortage of publicly available resources, such as software packages or calculation modules. Recently, there are a lot of social services enough for such data to be easily published, however, it is still not easy to reproduce and verify their results computationally.

The author has impremented a calculation package for Python language ([7]) named “SetOptTools.” This package includes computational functions for set relations between sets represented by systems of linear inequalities such as $\{\boldsymbol{x} | A\boldsymbol{x} \leq \boldsymbol{b}\}$. The system adopts sublinear characterization methods proposed in ([5]) to evaluate “set relations” defined in [1]. Moreover, characterization functionals have been studied to quantify set relations (e.g., [8, 9, 10, 11]). This research is motivated by characterization theorems (e.g., [8]), which establish the equivalence of set relations and the negativity of characterizing scalars.

For imprementation, this module consists of four different subsystems. Each function requires the compared sets and a convex ordering cone represented as the intersection of finitely many half spaces. These subsystems independently scalarize set inequalities by checking the consistency and redundancy of given inequalities and the optimality of evaluation problems as follows.

The first one computes strict Type 1 set relations, which can be estimated by a series of linear maximization problems. The second one handles stronger Type 2U and 2L relations, which are obtained by a combination of linear maximization and minimization problems. On the other hand, pre-ordering Type 3U and 3L require several steps: decomposing the compared sets into a bounded region (convex hull) and unbounded directions (rays); checking some algebraic geometry related to the unbounded directions of the given sets and the cone; computing the exact values of characterization functionals. The last one deals with the weakest Type 4 relations by solving several linear programming problems.

In this talk, we first introduce SetOptTools and its technical concepts. Next, we consider some numerical examples of evaluating set inequalities using the package.

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A MINIMAX THEOREM FOR SET-VALUED MAPS AND ITS APPLICATION TO MULTI-OBJECTIVE UNCERTAINTY

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In this presentation, we propose two-person games with set payoffs. Then we introduce several concepts of saddle points and equilibrium points of two-person games with set payoffs. We also present an existence theorem of saddle-point by using Kakutani-Glicksberg-Fan's fixed point theorem[5]. We also present a new type minimax theorem for set-valued maps[1]. The proposed theory described above can be interpreted as uncertain multi-objective game theory. We also formulate an uncertain multi-objective game.

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PROXIMAL POINT ALGORITHM ON GEODESIC SPACES WITH A NEW RESOLVENT OPERATOR

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The proximal point algorithm is one of the most useful methods for finding a minimizer of a given convex function. It was introduced by Rockafellar [8] in the setting of Hilbert spaces. In this method, resolvents of a convex function play an important role in approximating minimizers of a given convex function. In a complete CAT(0) space X, a resolvent for a convex function f

is defined by

$$J_f x = \operatorname*{argmin}_{y \in X} \left(f(y) + \frac{1}{2} d(y,x)^2 \right)$$

for $x \in X$, see [7]. Using this resolvent, Bačák [1] investigated the proximal point algorithm in complete CAT(0) spaces and proved that the sequence $\{J_{\lambda_n f} x_n\}$ is Δ-convergent to a minimizer of f. In 2016, Kimura and Kohsaka [2] introduced the following resolvent in an admissible complete CAT(1) space:

$$R_f x = \operatorname*{argmin}_{y \in X} \left(f(y) + \tan d(y,x) \sin d(y,x) \right)$$

for $x \in X$. On the other hand, Kimura and the present author [3] proposed another type of resolvent defined by

$$S_f x = \operatorname*{argmin}_{y \in X} \left(f(y) - \log(\cos d(y,x)) \right)$$

in an admissible complete CAT(1) space.

In 2024, Kimura and Sudo [5] established a fixed point theory on Banach spheres equipped with a convex structure. They also introduced a resolvent of a convex function f having a minimizer on a Banach sphere X by

$$R_f x = \operatorname*{argmin}_{y \in X} \left(f(y) - \langle y, Jx \rangle \right)$$

for $x \in X$, and investigated the proximal point algorithm associated with this resolvent, see [6].

In this talk, we investigate the proximal point algorithm generated by a new resolvent in geodesic spaces.

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A FRAMEWORK OF SMOOTHING PENALTY METHODS FOR CONVEX SECOND-ORDER CONE PROGRAMMING: FUNDAMENTALS AND APPLICATION

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Penalty and barrier methods have been extensively developed as fundamental techniques in optimization theory. Against this background, primal-dual interior-point methods have emerged as highly efficient algorithms. Currently, many commercial solvers for Second-Order Cone Programming (SOCP) and Semidefinite Programming (SDP) rely on these interior-point methods, making them undeniably central to modern optimization applications. However, there remain specific scenarios where primal-dual interior-point methods struggle. In this presentation, we propose a novel smoothing penalty method utilizing asymptotic cones. We will also explore its practical applications to Model Predictive Control.

Here, we focus on the general convex second-order cone programming (CSOCP) problem. Let $\mathcal{K}$ denote the general second-order cone (SOC) in $\mathbb{R}^n$ given by

$$\mathcal{K} := \mathcal{K}^{n_1} \times \cdots \times \mathcal{K}^{n_r},$$

with $r, n_1, \ldots, n_r \geq 1$, $n_1 + \cdots + n_r = n$, and

$$\mathcal{K}^{n_i} := \left\{ (x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{n_i - 1} \;\middle|\; \|x_2\| \leq x_1 \right\},$$

where $\|\cdot\|$ denotes the Euclidean norm. Note that $\mathcal{K}^1$ denotes the set of nonnegative real numbers $\mathbb{R}_+$. In this presentation, we consider the following CSOCP problem:

$$\begin{cases} \min & f(\xi) \\ \text{s.t.} & A\xi - b \preceq_{\mathcal{K}} 0, \end{cases}$$

where A is an $n \times m$ matrix with $n \geq m$ and $\operatorname{rank}(A) = m$, $b \in \mathbb{R}^n$, $f : \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ is a closed proper convex function, and $x \preceq_{\mathcal{K}} 0$ means $-x \in \mathcal{K}$.

For simplicity, we focus on a single second-order cone $\mathcal{K}^n$, as the analysis in this setting can be naturally extended to the general second-order cone $\mathcal{K}$. In other words, we primarily investigate the following formulation:

$$\begin{cases} \min & f(\xi) \\ \text{s.t.} & A\xi - b \preceq_{\mathcal{K}^n} 0. \end{cases}$$

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